

## Discussion 12 Solution

### Topics

- Monopoly
- Price Discrimination
- Cournot Duopoly

### Problem 1

Verisson is a monopoly in the provision of broadband plans in Madison. Verisson's variable cost and marginal cost are given by  $VC = 4q$  and  $MC = 4$ , respectively. Verisson fixed cost is equal to \$1,500 (it requires a huge investment in infrastructure to be able to provide broadband plans). The demand for broadband plans in Madison is given by  $Q = 84 - P$ .

- a) Derive the TC for Verisson. Does Verisson's technology exhibit economies of scale?

$$TC = FC + VC. \text{ Then } TC = 1500 + 4q.$$

*Economies of scale refer to the observation that ATC is decreasing as Q increases (over a relevant range determined by demand). In this specific case,  $MC = 4$  is constant, so  $ATC = TC/q = 1500/q + 4$  always decreases as  $q$  increases. Thus, Verissons technology exhibits economies of scale.*

- b) Derive the Marginal Revenue (MR) of Verisson. Find the number of plans that Verisson provides and the price at which they sell them. Is Verisson making profits?

*To compute MR, you have to first express the demand in terms of P. So you get  $P = 84 - Q$ . Using the fact that MR has the same y-intercept and a slope that is twice as steep if the demand is linear, you get that  $MR = 84 - 2Q$ .*

*Using the profit-maximizing condition  $MC = MR$ , Verisson would choose QM such that  $MC = MR$ . Substituting the equations,  $4 = 84 - 2Q$ . Then  $Q = 40$ . If Verisson only provides 40 plans, they can sell them at a price  $P = 44$  (plugging in Q into the demand curve).*

*Profits = TR - TC =  $P*Q - 1500 - 4*Q = 44*40 - 1500 - 4*40 = 100$ . Verisson is making positive economic profits.*

- c) Compute the CS and PS in the market. What is the DWL of this monopoly?

*Graphing MR, MC and D it is easy to verify that  $CS = 40*40/2 = 800$ , and  $PS = 1,600$ . If the market was efficient (for example, in a perfectly competitive market)  $MC = P$ . Thus  $Q^* = 80$ . The DWL is  $40*40/2 = 800$ .*

A rival firm Chartel, that has the same cost functions as Verisson, is considering whether to enter the market or not. If Chartel enters the market, demand is equally split between both firms. Thus, the demand for broadband plans that Chartel would face in Madison is given by  $Q = 42 - 0.5P$ .

- d) Derive the MR for Chartel. Find the number of broadband plans that Chartel would offer and the price at which it would offer those plans if it decides to enter.

*If Chartel decides to enter, then the demand it faces is half of the market demand. Then express  $P$  in terms of  $Q$ , we have the demand for Chartel as:  $P = 84 - 2Q$ , so  $MR = 84 - 4Q$ .*

*Profit maximizing requires  $MR = MC$ , so you get that  $84 - 4Q = 4$ , so  $Q = 20$  and  $P = 44$ . Profits =  $TR - TC = 44 * 20 - 1500 - 4 * 20 = -700$  ;0.*

- e) Would Chartel enter the market of broadband plans in Madison?

*Chartel would not enter the market because it would have negative profits. This exercise gives you intuition of why in some markets with very high fixed costs, it is "natural" that monopolies exist.*

Assume that Chartel doesn't enter the market, and that Verisson finds a way to identify the willingness to pay for plans of every single person in Madison by spying on the internet usage of its customers. Suppose that Verisson can charge different prices to different costumers.

- f) Find the number of plans that Verisson would provide.

*In other words, Verisson is now a monopoly that can first-degree price discriminate. Verisson's MR will be instead equal to the demand curve. [Intuition: If Verisson increases the quantity produced by 1, its revenue will increase by exactly the willingness to pay of this individual (this is the definition of the demand curve)].*

*Thus,  $MR = 84 - Q$ , so setting  $MR=MC$  we will get  $Q=80$ . This is also the efficient number of plans.*

- g) Compute CS, PS, and DWL. Compare and explain your results with the numbers in c.

*Verisson will extract all the CS. There is no DWL, because  $Q=80$  equals the quantity that would occur in a perfect competitive market, which is effective. PS will now be given by  $80 * 80 / 2 = 3200$ . Under first-degree price discrimination, there is no DWL.*

Suppose now that the local government only lets Verisson to charge different prices to people under 35 years and people with 35 years or more in the following year. Fixed costs are now zero ( $FC = 0$ ) but  $MC = 4$  as before.

Verisson estimates that the demand for broadband plans by age group is the following:

- D of people with less than 35 years:  $Q = 64 - P$
- D of people with 35 years or more:  $Q = 44 - P$

- h) Find the price that Verissson would charge to each of the different age groups if the firm can identify the age of the customers. How many plans would it sell to each group?

*In g., Verissson can discriminate between two different types of groups (third-degree price discrimination). It can charge two different prices depending on the group considered. To maximize profits, you have to set  $MC = MR$  separately for each demand curve.*

*For people with less than 35 years,  $MR_1 = 64 - 2Q$  and  $MC = 4$ . Using the profit-maximizing condition  $MR_1 = MC$ , we have  $Q_1 = 30$ , and plugging this quantity in the demand curve for people with less than 35 years you compute that Verissson can charge a price of  $P_1 = 34$ .*

*Considering separately the market for people with 35 years or more,  $MR_2 = 44 - 2Q$  and  $MC = 4$ . With the profit-maximizing condition  $MR_2 = MC$  you can find again that  $Q_2 = 20$  and  $P_2 = 24$ . Notice that Verissson charges a higher price to that group that is willing to pay more for the services (group with less than 35 years).*

- e) Compute CS for each age group and Verissson's profits. What is the DWL of the monopoly?

*For people with less than 35 years,  $CS_1 = 0.5 * (64 - 34) * 30 = 450$ , and profits= $(34-4)*30=900$ . To find the DWL, first find the efficient quantity and price, which is given by setting  $P=64-Q=MC$ , so efficient quantity is 60 and price is 4. So  $DWL_1 = 0.5 * (60 - 34) * (34 - 4) = 450$ .*

*For people with more than 35 years,  $CS_2 = 0.5 * (44 - 24) * 20 = 200$ , and profits= $(24-4)*20=400$ . To find the DWL, first find the efficient quantity and price, which is given by setting  $P=44-Q=MC$ , so efficient quantity is 40 and price is 4. So  $DWL_2 = 0.5 * (40 - 20) * (24 - 4) = 200$ . The DWL of the monopoly can be computed by the DWL on each of the submarkets.  $DWL = DWL_1 + DWL_2 = 650$ .*

## Problem 2

Consider the market for central processing units (CPUs), a key component in modern computers. This market consists of two firms: Intel and AMD. For simplicity, assume that both Intel and AMD have identical cost structures, where  $MC = AC = 30$  (we would change this later) for each firm. On any given day, the market demand for CPUs is given by  $P = 120 - Q$ .

- (a) Suppose the market for CPUs was controlled by a monopoly with the same cost structure as Intel and AMD. How many CPUs would this monopoly produce (call this  $Q_M$ ), and what price would it charge  $P_M$ ? *Solution*

*Solving the monopolist's problem in the usual way yields*

$$MR = 120 - 2Q = 30 \Rightarrow Q_M = 45$$

*Plugging into the demand function*

$$P_M = \$75$$

- (b) Suppose instead the market for CPUs was perfectly competitive, with every firm having the same cost structure as Intel and AMD. What would be the market equilibrium quantity  $Q_{PC}$  and price  $P_{PC}$ ? *Solution:*

*Solving the perfectly competitive firm's problem  $P = 30$*

$$120 - Q = 30 \implies Q_{PC} = 90.$$

- (c) Now return to reality, where Intel and AMD compete as Cournot duopolist. What is the reaction function of Intel? What is the reaction function of AMD? *Solution:*

*The reaction function for Intel is  $q_{Intel} = 45 - \frac{1}{2}q_{AMD}$  and  $q_{AMD} = 45 - \frac{1}{2}q_{Intel}$*

- (d) Find the quantity produced by each firm in a Cournot equilibrium,  $q_{Intel}^*$  and  $q_{AMD}^*$ . Then find the market quantity  $Q_C$  and market price  $P_C$  under this Cournot duopoly. *Solution:*

*Either by finding the intersection of the firms reaction functions or by using the Trick, we can see that  $q_{Intel}^* = q_{AMD}^* = \frac{2}{3}Q_M = \frac{2}{3}40 = 30$ . Therefore,  $Q_C = 60$  and  $P_C = 60$*

- (e) Compare the three industrial structures: monopoly, Cournot duopoly, and perfect competition. Rank these in terms of firms profits and the welfare of consumers (Hint: there is no need to calculate anything here. Use your intuition to rank these by comparing prices and quantities only.) *Solution:*

*In terms of firms profits, profits for monopoly > profits for a firm in Cournot > profits for a perfectly competitive firm = 0, since marginal cost equals average cost and is constant. In terms of consumer surplus, CS under perfect competition > CS under Cournot > CS under monopoly.*