Discussion 1 Solutions

Important Topics

- Math Review
- Opportunity Costs and Comparative Advantage (if time)

Math

We'll use a lot of algebra and graphs in this course. It's important that the rote math steps are automatic so we can focus on the more interesting economic applications. So, you should be comfortable graphing equations, finding an equation from a graph, solving for two unknowns with two equations, and calculating percentages.

Graphing a line is usually most conveniently done if we first have the equation in *slope intercept* form.

Slope intercept form: $y = mx + b$

- m is the slope, rise over run.
- b is the y-intercept.

Sometimes you will instead encounter the same equation, but in the form $x = a + cy$. We have x in terms of y instead of in the usual slope intercept form. Here, it turns out a is the x-intercept and c is the inverse slope.

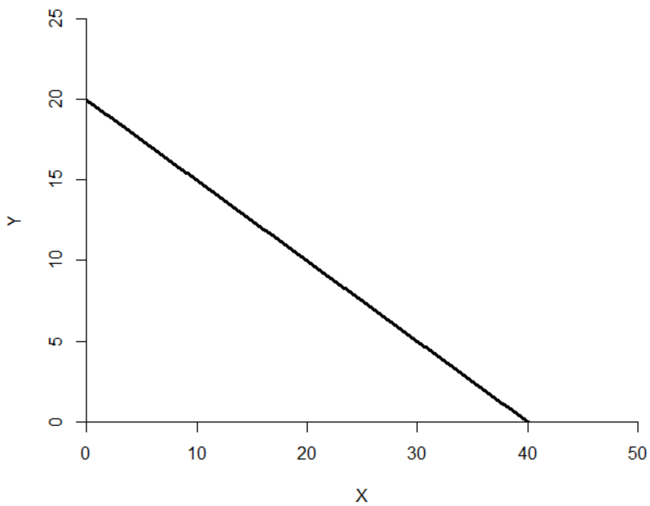
$$c = \frac{1}{m} \qquad a = \frac{-b}{m}$$

You can also ignore this relationship and just convert everything to slope intercept form and work from there.

Percentage change: If we have an original and new values, the percentage change is calculated as

$$\frac{\text{New Value} - \text{Old Value}}{\text{Old Value}} \times 100.$$

Exercise 1 Find the equation for the graph below. What is the area below the curve (the triangle formed with the axes)?



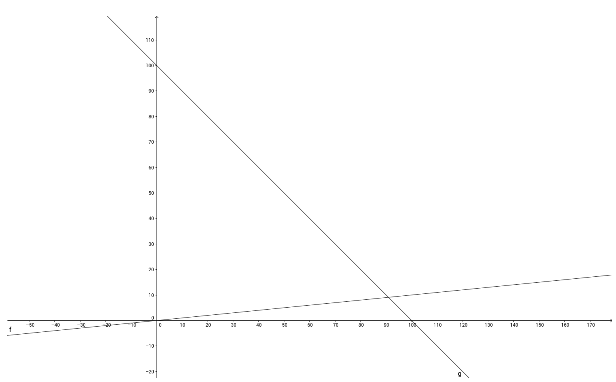
Solution: We see the y-intercept is 20. When we have both intercepts the slope will be $-\frac{y\text{-}int}{x\text{-}int}$. *So, the equation is* $y = 20 - \frac{1}{2}x$.

The area is $\frac{1}{2} base \times height = \frac{1}{2} 40 \times 20 = 400$.

Exercise 2 Graph $x = 10y$ and $y = 100 - x$.

Solution:

You might rearrange $x = 10y$ *as* $y = \frac{1}{10}x$ *to observe that the y-intercept is at zero and the slope is* $\frac{1}{10}$,



Exercise 3 Solve for x and y given the equations

$$y = 2x + 4y + 2$$

$$x = 2y + 4.$$

Solution: We can simplify the first equation to

$$-2x = 3y + 2$$

and then, dividing by negative two,

$$x = -\frac{3}{2}y - 1.$$

Now we can substitute this into the second equation,

$$x = -\frac{3}{2}y - 1 = 2y + 4.$$

Simplifying,

$$-5 = \frac{7}{2}y$$

$$y = -10/7$$

. *If* $y = -10/7$*, we get* $x = -20/7 + 4 = \frac{8}{7}$.

Exercise 4 In 2013, Theranos, the health technology company, was valued at \$9 billion. More recently, the company's value has been revised to \$800 million. What is the percentage change in the company value?

Solution: Luckily, we can ignore all the zeroes that come along with billions and millions if we recognize that 800 *million is .8 of one billion. Therefore, the percentage change can be calculated as*

$$\frac{.8 - 9}{9} \times 100 = \frac{-8.2}{9} \times 100 = \frac{-820}{9}.$$

This reduces to $-91\frac{1}{9}\%$. *The company lost over* 91% *of its value.*

Exercise 5 On June 14, 2000, the Indiana Pacers lost to the Los Angeles Lakers in game four of the NBA finals, giving the Lakers a 3-1 edge on the series. Shaquille O'Neal made 10 free throws on 17 attempts. Reggie Miller made 11 free throws on 12 attempts. How many more free throws would Shaq have to make in a row to match Reggie's percentage?

Solution: Here, the challenge is mostly in setting up the correct problem.

After Shaq shoots x more free throws, realize his percentage will be

$$\frac{10+x}{17+x} \times 100.$$

Reggie Miller's percentage is fixed at

$$\frac{11}{12} \times 100$$

. We don't have to simplify these any further to solve, just set up the following inequality,

$$\frac{10+x}{17+x} \times 100 \geq \frac{11}{12} \times 100.$$

We can get rid of the 100s on both sides,

$$\frac{10+x}{17+x} \geq \frac{11}{12}.$$

Then cross-multiply,

$$120 + 12x \geq 187 + 11x$$
$$x \geq 67.$$

So Shaq needs to hit at least 67 *consecutive free throws to match Reggie Miller's percentage.*

Economics

Exercise 6 A Robinson Crusoe Economy

Crusoe finds himself stranded on an island. He devotes 10 hours each day to either gathering coconuts or catching fish. Crusoe can gather 2 coconuts in an hour, but he needs 5 hours to catch a single fish. On the other side of the island is Friday. Friday also devotes 10 hours each day to gathering coconuts and fishing. He can gather 3 coconuts per hour and needs only 2.5 hours to catch a fish.

a.) What is the opportunity cost of gathering a coconut for Crusoe? For Friday?

Solution: See part b.)

b.) What is the opportunity cost of catching a fish for Crusoe? For Friday?

Solution: One coconut costs Crusoe $\frac{1}{10}$ fish. One coconut costs Friday $\frac{2}{15}$ fish. One fish costs Crusoe 10 coconuts. One fish costs Friday 7.5 coconuts.

It may be helpful to organize this information in a table.

maximum output

	fish	*coconuts*
Crusoe	*2*	*20*
Friday	*4*	*30*

opportunity cost

	fish	*coconuts*
Crusoe	*10 coconuts*	$\frac{1}{10}$ *fish*
Friday	$\frac{15}{2}$ *coconuts*	$\frac{2}{15}$ *fish*

c.) Draw Crusoe's production possibilities frontier with coconuts on the x-axis. Give its equation. On a separate graph, do the same for Friday.

Solution:

Note: "Assume constant opportunity cost" is the magic phrase that ensures we're working with linear PPFs here. Given that, you may find it easiest to start with two points on the PPF, and figure out the equation of the line that passes through them. The easiest two points to start with will usually be the extremes, all fish and no coconuts or all coconuts and no fish. This method is used below.

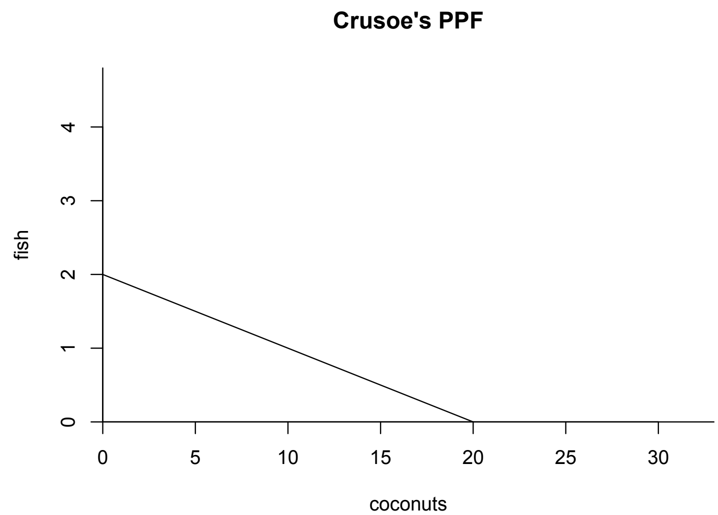
Crusoe can gather 20 coconuts if he devotes all of his time to gathering coconuts. He will catch only two fish if he spends all of his time fishing. The math is shown formally below, though it is straightforward.

$$\textit{Fish per day} = 10\ \frac{\textit{hours}}{\textit{day}} \div 5\frac{\textit{hours}}{\textit{fish}} = 2\frac{\textit{fish}}{\textit{day}}$$

$$\textit{Coconuts per day} = 10\frac{\textit{hours}}{\textit{day}} \times 2\frac{\textit{coconuts}}{\textit{hour}} = 20\frac{\textit{coconuts}}{\textit{day}}$$

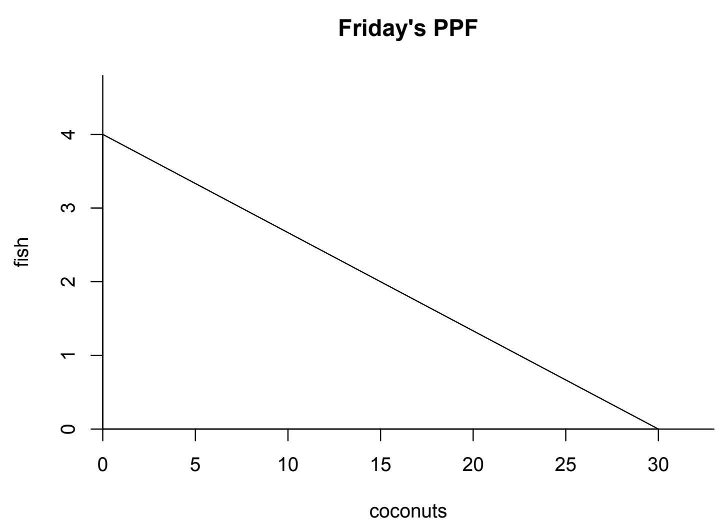
On our plot, this corresponds to points (0 coconuts, 2 fish) and (20 coconuts, 0 fish). The line that connects them will have a y-intercept of 2 and a slope of $-\frac{1}{10}$. This gives the PPF equation

$$y = 2 - \frac{1}{10}x$$



We can do the same for Friday. Note Friday can gather 30 coconuts per day if he devotes no time to fishing. Friday will catch 4 fish if he only fishes. This gives us the below PPF and the equation.

$$y = 4 - \frac{2}{15}x$$



One day, Crusoe and Friday befriend each other and they decide to work together.

d.) Who should specialize in coconuts? In fish? Justify your answer using the language of comparative advantage.

Solution:

Friday has the lowest opportunity cost in fish. Crusoe has the lowest opportunity cost in coconuts. Therefore, Friday has the comparative advantage in fish and Crusoe in coconuts. To achieve productive efficiency, Crusoe should specialize in coconuts and Friday should specialize in fish.

e.) Draw a new joint PPF (reflecting their combined production possibilities).

Solution:

There are three key points to drawing a joint PPF. We consider three extreme cases: everyone makes only good y, everyone makes only good x, and everyone specializes 100% so that each person spends all of their time producing the good in which they have a comparative advantage. The joint PPF will connect these points.

Our three extreme points are (0,6), (50,0), and (20,4).