Discussion 4 Solution

Topics

- Elasticities

Problems

1. Consider the markets for football game tickets and the market for basketball game tickets. The demand curve for the former is $P + Q = 100$. But all we know about the demand for basketball game tickets is that no one is willing to buy any ticket when the price is above \$80 per ticket.

   a) Find the price elasticity of demand for both the markets separately at the price of \$50 per ticket.

   We are asked to calculate the elasticity of demand at a particular price, so the point elasticity formula should be applied here,

   $$E_d = \frac{1}{m} \cdot \frac{P}{Q}$$

   where m is the slope of the demand curve.

   Given the demand curve for football tickets market, we can see that the slope=-1, and that at $P = 50$, the corresponding $Q = 50$. Applying the formula above, the point elasticity of demand for the football ticket market is $E_d = -\frac{50}{50} = -1$.

   As for the basketball tickets, we do not have information on the slope of the demand curve.The only clue is when $P = 80$, $Q = 0$. This is the intercept on the price axis! Now let me show you that we can find the point elasticity using only the intercept:

   Plugging the intercept into the demand curve, we have $P = mQ + 80$, we can re arrange it and write it as $mQ = P - 80$. Recall that in the point elasticity formula, the denominator is exactly mQ! So now we can express mQ in terms of P, which means that the whole point elasticity can be expressed by price only. To put it formally,

   $$E_d = \frac{1}{m} \cdot \frac{P}{Q} = \frac{P}{mQ} = \frac{P}{P - 80} = \frac{50}{50 - 80} = -\frac{5}{3}$$

   b) How do prices need to change from \$50 per ticket in order to increase revenues in both markets separately?

   In part (a), we find that the football tickets market price \$50 corresponds to the unit elastic part of the demand curve, so the revenue is already maximized.

   However for the basketball tickets market, the absolute value of the elasticity is $\frac{5}{3}$, larger than 1, suggesting that we are on the elastic part now. Hence prices need *to go down in order to increase revenue. (In particular price needs to go down to* 40 *per ticket to maximize revenue.)*

c) How do the elasticities at price of \$50 differ in the two markets if instead of the maximum willingness to pay for basket ball is \$100 instead of \$80.

In this case, the y-axis intercept changes to 100 *instead of* 80. *Using the last formula in (a),*

$$E_d = \frac{1}{m} \cdot \frac{P}{Q} = \frac{P}{mQ} = \frac{P}{P-100} = \frac{50}{50-100} = -1$$

So we can see that the elasticities of demand for basket ball tickets is -1*, the same as football tickets.*

OBSERVATION: Markets with linear demand curves and the same y-axis intercepts have the same elasticities at a given price, no matter how the slopes differ.

2. The following table gives part of the demand schedule for widgets in USA:

Price	1	3	5
Quantity Demanded	18	10	4

a) Calculate the price elasticity of demand as the price increases from \$1 to \$3 using the mid-point formula. What happens to the total revenue as a result of the price increase?

Using the midpoint elasticity formula, we have :

$$E_d = \frac{\frac{10-18}{(10+18)/2}}{\frac{3-1}{(3+1)/2}} = -\frac{4}{7}$$

Therefore, $|E_d|$ *is less than* 1*, so demand is inelastic on this portion of the demand curve. Hence, an increase in price will increase total revenue.*

b) Calculate the price elasticity of demand as the price increases from \$3 to \$5 using the mid-point formula. What happens to the total revenue as a result of the price increase?

Using the midpoint elasticity formula, we have :

$$E_d = \frac{\frac{4-10}{(4+10)/2}}{\frac{5-3}{(5+3)/2}} = -\frac{12}{7}$$

Therefore, $|E_d|$ *is larger than* 1*, so demand is elastic on this portion of the demand curve. Hence, a decrease in price will increase total revenue.*

3. Suppose demand for Donuts in Madison is given by $P = 120 - 3Q$

a) Calculate the price elasticity of demand at $P = \$30$. What adjustment should be made to the price to increase total revenue?

Plugging $P = 30$ into the demand equation gives $Q = 30$. We can now calculate the elasticity at $P = 30$ using the slope formula,

$$E_d = \frac{1}{m} \cdot \frac{P}{Q} = -\frac{1}{3} \cdot \frac{30}{30} = -\frac{1}{3}$$

Since $|E_d|$ is less than 1, the demand is inelastic at $P = 30$. Therefore, total revenue should increase to achieve a higher level (but not higher than the price at which the elasticity is 1).

b) Calculate the price elasticity of demand at $P = 90$. What adjustment should be made to the price to increase total revenue?

Plugging $P = 90$ into the demand equation gives $Q = 10$. We can now calculate the elasticity at $P = 90$ using the slope formula,

$$E_d = \frac{1}{m} \cdot \frac{P}{Q} = \frac{1}{3} \cdot \frac{90}{10} = -3$$

Since $|E_d|$ is larger than 1, the demand is elastic at $P = 90$. Therefore, total revenue should decrease to achieve a higher level (but not lower than the price at which the elasticity is 1).

c) Calculate the price and quantity at which the total revenue is maximized. What is the maximum revenue that can be obtained?

The total revenue is maximized at the point where $|E_d|$ = 1 or , which E_d = -1 . Plugging in the slope formula for E_d, we have

$$E_d = \frac{1}{m} \cdot \frac{P}{Q} = -\frac{1}{3} \cdot \frac{P}{Q} = -1$$

And don't forget we have demand curve, which gives us the relationship between P and Q! Therefore we can plug in $P = 120 - 3Q$ into the equation above, and get an equation in terms of Q only :

$$-\frac{1}{3} \cdot \frac{P}{Q} = -\frac{1}{3} \cdot \frac{120 - 3Q}{Q} = -1$$

Thus $Q = 20$.