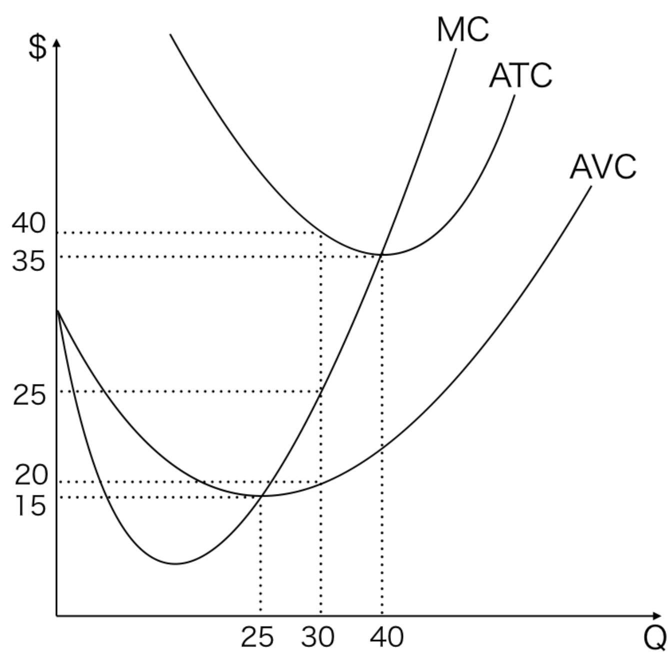
Discussion 8: Solutions

Important Topics

- Cost Curves
- Profit Maximization
- Perfect Competition
- Shut-down/Break-Even Prices
- Supply Curve Derivation

Exercise

Exercise 1 William's winery has cost functions illustrated below, where Q denotes gallons of wine. Suppose that the wine market is perfectly competitive.



(a) Find the break-even price and the shut-down price.

(b) Draw the short-run supply curve on the graph.

(c) Calculate the fixed cost.

(d) If the price is $P = 25$, then what is William's profit?

Solution

(a) The break-even price is determined at the intersection point of ATC and MC:

$$P_{be} = 35.$$

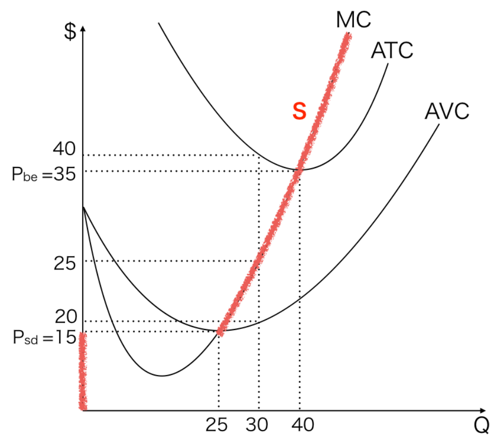
The shut-down price is determined at the intersection point of AVC and MC:

$$P_{sd} = 15.$$

(b) If the price is below the shut-down price $P_{sd} = 15$, then William produces nothing. Otherwise, by the price-taker and profit-maximization conditions,

$$P = MR = MC,$$

so the supply-curve coincides with the graph of MC function.



(c) At $Q = 30$,

$$AFC = ATC(30) - AVC(30) = 40 - 20 = 20.$$

Hence,

$$FC = AFC \times Q = 20 \times 30 = 600.$$

(d) Given $P = 25$, William produces $Q = 30$. Hence, the profit is

$$\begin{aligned}\pi(30) &= TR(30) - TC(30) \\ &= P \times Q - ATC \times Q \\ &= 25 \times 30 - 40 \times 30 \\ &= -450.\end{aligned}$$

Exercise 2 Consider a bakery with the following cost functions in a perfectly competitive bread market:

$$\begin{aligned} TC &= q^2 + 4q + 5, \\ MC &= 2q + 4, \end{aligned}$$

where q denotes pounds of bread.

(a) What is the shut-down price for this bakery?

(b) Derive the short-run supply curve.

Suppose that there are 10 bakeries with the same cost functions in the bread market and that the market demand curve is given by $P = -Q_d + 40$.

(c) Find the short-run equilibrium in the bread market.

Solution

(a) Note that

$$AVC = \frac{VC}{q} = \frac{q^2 + 4q}{q} = q + 4.$$

Hence, at the shut-down price,

$$2q + 4 = MC = AVC = q + 4,$$

so $q = 0$. By plugging this back into MC function, the shut-down price is

$$P_{sd} = 2 \times 0 + 4 = 4.$$

(b) The supply curve is

$$\begin{cases} P = MC = 2q + 4 & \text{if } P \geq 4, \\ q = 0 & \text{if } P < 4. \end{cases}$$

(c) Note that for each $P \geq 4$, the optimal output level for an individual bakery is

$$q = \frac{P}{2} - 2.$$

Since all the 10 firms have the same cost function, they have the same optimal output level. Hence, the market supply curve is given by

$$Q_s = \begin{cases} 10q = 10(\frac{P}{2} - 2) = 5P - 20 & \text{if } P \geq 4, \\ 0 & \text{if } P < 4. \end{cases}$$

On the other hand, the market demand is

$$Q_d = 40 - P.$$

Hence, at the equilibrium,

$$5P^* - 20 = Q_s = Q_d = 40 - P^*,$$

so

$$P^* = \frac{60}{6} = 10.$$

By plugging this into the market demand curve,

$$Q^* = 40 - 10 = 30.$$

Therefore, the equilibrium is $(Q^*, P^*) = (30, 10)$.

Exercise 3 You are working in a small construction company in a perfectly competitive housing market. You accidentally spilled coffee on an important file below, which summarizes the company's costs.

Q	TC	VC	FC	ATC	AVC	AFC	MC
0				—	—	—	—
1							**50**
2		**200**					
3				**600**			
4					**600**		
5	**5000**					**120**	

(a) Restore the file by filling all the blanks.

(b) If the market price is $P = 1200$, then what is the profit of the construction company?

Solution.

(i)

Q	TC	VC	FC	ATC	AVC	AFC	MC
0	600	600	600	—	—	—	—
1	650	50	600	650	50	600	**50**
2	800	**200**	600	400	100	300	150
3	1800	1200	600	**600**	400	200	1000
4	3000	2400	600	750	**600**	150	1200
5	**5000**	4400	600	1000	880	**120**	2000

Step 1. First, since $AFC = 120$ at $Q = 5$,

$$FC = AFC \times Q = 120 \times 5 = 600.$$

FC is independent of output levels, and thus we can fill in all the blanks for FC with 600.
Second, for each output level Q,

$$AFC = \frac{FC}{Q} = \frac{600}{Q}.$$

Third, at $Q = 0$,

$$VC(0) = 0,$$
$$TC(0) = VC(0) + FC = 600.$$

Q	TC	VC	FC	ATC	AVC	AFC	MC
0	600	0	600	—	—	—	—
1			600			600	**50**
2		**200**	600			300	
3			600	**600**		200	
4			600		**600**	150	
5	**5000**		600			**120**	

Step 2. At $Q = 1$,

$$TC(1) = TC(0) + MC(1) = 600 + 50 = 650,$$
$$VC(1) = TC(1) - FC = 650 - 600 = 50.$$

Hence,

$$ATC(1) = \frac{TC(1)}{1} = \frac{650}{1} = 650,$$
$$AVC(1) = \frac{VC(1)}{1} = \frac{50}{1} = 50.$$

Q	TC	VC	FC	ATC	AVC	AFC	MC
0	600	0	600	—	—	—	—
1	650	50	600	650	50	600	**50**
2		**200**	600			300	
3			600	**600**		200	
4			600		**600**	150	
5	**5000**		600			**120**	

Step 3. At $Q = 2$,

$$TC(2) = VC(2) + FC = 200 + 600 = 800,$$
$$MC(2) = TC(2) - TC(1) = 800 - 650 = 150.$$

Hence,

$$ATC(2) = \frac{TC(2)}{2} = \frac{800}{2} = 400,$$
$$AVC(2) = \frac{VC(2)}{2} = \frac{200}{2} = 100.$$

Q	TC	VC	FC	ATC	AVC	AFC	MC
0	600	0	600	—	—	—	—
1	650	50	600	650	50	600	**50**
2	800	**200**	600	400	100	300	150
3			600	**600**		200	
4			600		**600**	150	
5	**5000**		600			**120**	

Step 4. At $Q = 3$,

$TC(3) = ATC(3) \times 3 = 600 \times 3 = 1800.$

Hence,

$$VC(3) = TC(3) - FC = 1800 - 600 = 1200,$$
$$AVC(3) = \frac{VC(3)}{3} = \frac{1200}{3} = 400,$$
$$MC(3) = TC(3) - TC(2) = 1800 - 800 = 1000.$$

Q	TC	VC	FC	ATC	AVC	AFC	MC
0	600	0	600	—	—	—	—
1	650	50	600	650	50	600	**50**
2	800	**200**	600	400	100	300	150
3	1800	1200	600	**600**	400	200	1000
4			600		**600**	150	
5	**5000**		600			**120**	

Step 5. At $Q = 4$,

$VC(4) = AVC(4) \times 4 = 600 \times 4 = 2400.$

Hence,

$$TC(4) = VC(4) + FC = 2400 + 600 = 3000,$$
$$ATC(4) = \frac{TC(4)}{4} = \frac{3000}{4} = 750,$$
$$MC(4) = TC(4) - TC(3) = 3000 - 1800 = 1200.$$

Q	TC	VC	FC	ATC	AVC	AFC	MC
0	600	0	600	—	—	—	—
1	650	50	600	650	50	600	**50**
2	800	**200**	600	400	100	300	150
3	1800	1200	600	**600**	400	200	1000
4	3000	2400	600	750	**600**	150	1200
5	**5000**		600			**120**	

Step 6. At $Q = 5$,

$VC(5) = TC(5) - FC = 5000 - 600 = 4400.$

Hence,

$$ATC(5) = \frac{TC(5)}{5} = \frac{5000}{5} = 1000,$$
$$AVC(5) = \frac{VC(5)}{5} = \frac{4400}{5} = 880,$$
$$MC(5) = TC(5) - TC(4) = 5000 - 3000 = 2000.$$

Q	TC	VC	FC	ATC	AVC	AFC	MC
0	600	0	600	—	—	—	—
1	650	50	600	650	50	600	**50**
2	800	**200**	600	400	100	300	150
3	1800	1200	600	**600**	400	200	1000
4	3000	2400	600	750	**600**	150	1200
5	**5000**	4400	600	1000	880	**120**	2000

(b) By the price-taker and profit-maximization conditions,

$$P = MR = MC.$$

Hence, at $P = 1200$, the company will produce $Q = 4$. Therefore, the profit is

$$\pi = TR - TC = 1200 \times 4 - 3000 = 1800.$$