Handout-Week 13

Topics

- Cournot Duopoly
- Price Leadership
- Collusion

Cournot Duopoly

Consider the market for central processing units (CPUs), a key component in modern computers. This market consists of two firms: Intel and AMD. For simplicity, assume that both Intel and AMD have identical cost structures, where $MC = AC = 30$ (we would change this later) for each firm. On any given day, the market demand for CPUs is given by $P = 120 - Q$.

(a) Suppose the market for CPUs was controlled by a monopoly with the same cost structure as Intel and AMD. How many CPUs would this monopoly produce (call this Q_M), and what price would it charge P_M? **Solving the monopolist's problem in the usual way yields**

$$\mathbf{MR = 120 - 2Q = 30 \Rightarrow Q_M = 45}$$

Plugging into the demand function

$$\mathbf{P_M = \$75}$$

(b) Suppose instead the market for CPUs was perfectly competitive, with every firm having the same cost structure as Intel and AMD. What would be the market equilibrium quantity Q_{PC} and price P_{PC}? **Solving the perfectly competitive firm's problem $\mathbf{P = 30}$**

$$\mathbf{120 - Q = 30 \Longrightarrow Q_{PC} = 90}.$$

(c) Now return to reality, where Intel and AMD compete as Cournot duopolist. What is the reaction function of Intel? What is the reaction function of AMD? **The reaction function for Intel is $\mathbf{q_{Intel} = 45 - \frac{1}{2}q_{AMD}}$ and $\mathbf{q_{AMD} = 45 - \frac{1}{2}q_{Intel}}$**

(d) Find the quantity produced by each firm in a Cournot equilibrium, q^*_{Intel} and q^*_{AMD}. Then find the market quantity Q_C and market price P_C under this Cournot duopoly. **Either by finding the intersection of the firms reaction functions or by using the Trick, we can see that $\mathbf{q^*_{Intel} = q^*_{AMD} = \frac{2}{3}Q_M = \frac{2}{3}40 = 30}$. Therefore, $Q_C = 60$ and $P_C = 60$**

(e) Compare the three industrial structures: monopoly, Cournot duopoly, and perfect competition. Rank these in terms of firms profits and the welfare of consumers (Hint:there is no need to calculate anything here. Use your intuition to rank these by comparing prices and quantities only.) **In terms of firms profits, profits for monopoly > profits for a firm in Cournot > profits for a perfectly competitive firm = 0, since marginal cost equals average cost and is constant. In terms of consumer surplus, CS under perfect competition > CS under Cournot > CS under monopoly.**

(f) Suppose Intel's marginal cost is $MC = 20$. What's Intel's reaction function? **Given that $\mathbf{MC = 30}$ if Intel were a monopoly then $Q'_M = 50$. On the other hand if the firm were in a perfectly competitive market $\mathbf{Q'_{PC} = 100}$, the the reaction function is given by $\mathbf{q_{Intel} = 50 - \frac{1}{2}q_{AMD}}$** The reaction function for AMD is unchanged because the marginal cost of AMD is unchanged.

(g) Find the new quantity produced by each firm in a Cournot equilibrium, q^*_{Intel} and q^*_{AMD}. Can you use the $\frac{2}{3}$ rule. **There is no symmetry so, the equilibrium quantities aren't equal to $\frac{2}{3}$ the monopoly quantity. Moreover, $\mathbf{q^*_{AMD} = \frac{80}{3}}$ and $\mathbf{q^*_{Intel} = \frac{110}{3}}$**

Price Leadership

Consider the price leadership model of oligopoly in the market for tablet computers. Suppose the market demand for tablets is given by

$$P = 100 - \frac{1}{10}Q_M$$

Suppose the dominant firm, Apple, has a marginal cost function MC_D given by

$$MC_D = \frac{9}{10}Q_D + 2.5$$

Furthermore, suppose there are 20 other identical small firms that produce tablets, EACH with marginal cost function MC_{SF} given by

$$MC_{SF} = 2q_{SF} + 5$$

(a) Find the supply curve for small firms as a function of price. **First, one needs to solve $\mathbf{Q_{SF}}$ in terms of P, $\mathbf{MC_{SF}}$ to find each firm's supply curve. Doing so yields $\mathbf{q_{SF} = \frac{P-5}{2}}$. Since there are 20 firms, we can add quantities by multiplying by 20, resulting in**

$$\mathbf{Q_{SF} = 10P - 50}$$

(b) Find the residual demand function for the dominant firm, Q_D. (Make sure you find all 3 segments of this curve, yielding a piecewise demand function. The easiest way to do this is to find the two kink points. **From our equation for $\mathbf{Q_{SF}}$, we can see that $\mathbf{Q_{SF} = 0}$ only when $\mathbf{P = \$5}$. At prices below 5, residual demand is just market demand. The second kink point is at the price where the small firm supply equals the market demand. Solving for this price yields $\mathbf{P = \$52.50}$. So above this price, the quantity supplied by the dominant firm is 0. If price is between \$5 and \$52.50, then to find the dominant firm's residual demand we subtract the small firm's supply from market demand. That is, $\mathbf{Q_D = Q_M - Q_{SF} = (1000 - 10P) - (10P - 50) = 1050 - 20P}$. In summary,**

$$\begin{cases} 1000 - 10P & P \leq 5 \\ 1050 - 20P & 5 \leq P \leq 52.50 \\ 0 & 52.50 \leq P \end{cases}$$

(c) Solve for the quantity produced by the dominant firm in equilibrium, Q^*_D. **The dominant firm acts like a monopoly with respect to its residual demand. So first we must find its marginal revenue curve $\mathbf{MR_D}$. We can assume that he will**

produce a quantity that yields a price between \$5 and \$52.50. Inverting the relevant portion of the residual demand curve yields $\mathbf{P = 52.50 - \frac{1}{20}Q_D}$, which implies $\mathbf{MR_D = 52.50 - \frac{1}{10}Q_D}$. Setting $\mathbf{MR_D = MC_D}$ yields $\mathbf{Q_D^* = 50}$

(d) Find the equilibrium price P^*. **Plugging $\mathbf{Q_D^* = 50}$ into the dominant firm's residual demand curve in the relevant region yields $\mathbf{P^* = 50}$.**

(e) Find the quantity produced by the small firms in equilibrium Q^*_{SF} and the market quantity in equilibrium Q^*_M. **Plugging $\mathbf{P^* = 50}$ into the small firms' supply curve yields $\mathbf{Q^*_{SF} = 450}$, and plugging $\mathbf{P^* = 50}$ into the market demand curve yields $\mathbf{Q^*_M = 500}$. One can easily verify that $\mathbf{Q^*_M = Q^*_D + Q^*_{SF}}$**

Collusion

Suppose there are 3 countries that are the only countries in the world to produce oil. The market demand for oil is $P = 260 - 2Q$. Marginal cost is \$20 on each unit sold.

(a) Suppose they choose to collude. They first figure out how to maximize total profits and then divide the production and profits between them. How much do they each produce in this scenario? What profits does each earn? **To maximize total profits, they must produce the quantity that a monopolist would produce between the 3 of them. Thus solve the monopolists problem: $\mathbf{MR = 260 - 4Q = MC = 20}$ which means $\mathbf{Q = 60}$ and $\mathbf{P = \$140}$ which means each produces 20 for a total revenue of \$2800. TC for each is $\mathbf{20 * \$20 = \$400}$ so each earns profits equal to \$2400.**

(b) Suppose one of the 3 countries cheats and produces an extra 20 barrels. What are the profits of the 3 countries in this scenario? Does it pay to be the cheater? **Total market $\mathbf{Q = 80}$ so now $\mathbf{P = \$100}$. The country that cheated is making 40 units so its revenue is \$4000. Total costs $\mathbf{= 40 * \$20 = \$800}$, so its profits when it cheats are \$3200. The other countries profits go down to $\mathbf{\$100 * 20 - \$20 * 20 = \$1600}$. Thus cheating benefits the cheater and hurts everyone else.**

(c) Now suppose all 3 countries cheat and produce the extra 20 barrels. What are the profits of each? **Market quantity $\mathbf{Q = 120}$ then $\mathbf{P = \$20}$. Since this is the same as MC, we dont have to calculate anything, all three countries are making zero profits!**