

Discussion 9 - Solutions

Important Topics

- Short and Long Run Equilibrium
- Input Markets

Quick Review

1 What is the fixed cost for a firm with costs $TC = 5 + \frac{5+5q+5q^2}{q+1}$?

Solution: $FC = TC(q = 0) = 10$

2 A firm should operate in the short run only when it can cover its ...

- Variable costs
- Fixed costs
- Total costs

Solution: Variable costs. In the short run, fixed costs should not affect decision-making. This rules out choice b and c.

Exercise 1 Find the short run supply curve for a firm with $TC = 10 + 10\sqrt{q} + 5q^2$ and $MC = 10q + \frac{5}{\sqrt{q}}$.

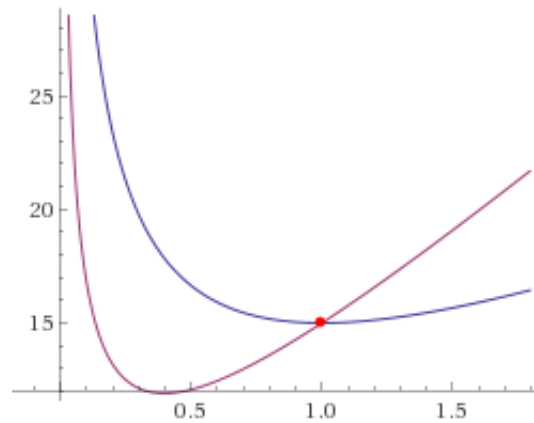
Solution: We know that a firm's short run supply curve is the MC curve above the shutdown price.

The shutdown price is found by solving $AVC = MC$.

$$\begin{aligned}
 TVC &= 10\sqrt{q} + 5q^2 \\
 AVC &= TVC/q = \frac{10}{\sqrt{q}} + 5q \\
 AVC = MC &\iff \frac{10}{\sqrt{q}} + 5q = 10q + \frac{5}{\sqrt{q}} \\
 &\iff \frac{5}{\sqrt{q}} = 5q \\
 &\iff \frac{1}{\sqrt{q}} = q \\
 &\iff q = 1
 \end{aligned}$$

Thus, the shutdown price is $P = 15$.

Now we have supply $P = 10q + \frac{5}{\sqrt{q}}$ for $P \geq 15$ and $q = 0$ for $P < 15$.



Exercise 2 Initially, the hoverboard market is served by 10 firms. Suppose all firms are identical and face the following costs and market demand:

$$MC = 10 + 10q$$

$$TC = 20 + 10q + 5q^2$$

$$Q^D = 30 - P$$

a.) What is the market supply curve? *Solution: We find the supply curve by aggregating the MC curves.*

Each firm sets $P = MC = 10 + 10q$. So,

$$q_{firm}^s = \frac{P - 10}{10}$$

and

$$Q_{market}^S = 10 \times q_{firm}^s = P - 10.$$

Here, the shutdown price is 10, so we say $Q_{market}^S = 0$ for $P \leq 10$ and $Q_{market}^S = P - 10$ else.

b.) Draw the usual graphs (demand, costs, etc) for the individual firm and the market. Label the break-even and shutdown prices. What is the short run equilibrium price and quantity?

Solution: Graph omitted. Breakeven price at $MC=ATC$. Shutdown price at $P=AVC$.

SR Equilibrium:

$$Q_{\text{market}}^S = Q^D$$

$$P - 10 = 30 - P \iff P = 20 \implies Q = 10$$

c.) How much does each firm produce? Will any firms exit in the short run?

Solution: (Dropping the subscript notation, we let q be the firm output and Q be the market output.)

Because the firms are identical, $Q = n \times q$ where n is the number of firms. Therefore with 10 firms, $q = 1$.

Firms will not exit because $TR = 20 > VC = 15$.

d.) Is this a long run equilibrium? Find the long run equilibrium price and quantity.

Solution: *This is not a LR equilibrium with $TR = 20 < TC = 35$. In the LR, $TR = TC \iff P = ATC$. Now we solve for this,*

$$P = MC = 10 + 10q = ATC = \frac{20 + 10q + 5q^2}{q} = \frac{20}{q} + 10 + 5q$$

$$5q = \frac{20}{q}$$

$$q^2 = \frac{20}{5} = 4 \implies q = 2.$$

Plug this into the MC curve to find the price,

$$P = 10 + 10(2) = 30.$$

e.) How many firms will be in the market in the long run? *Solution:*

We know that, in the LR, $P = 30$ and each firm will produce $q = 2$. Using $P = 30$, we know $Q^D = 30 - 30 = 0$. No transactions will occur, there will be zero firms. This wouldn't be the case if we made fixed costs lower or increased demand.

Exercise 3 The market for plastic chairs in Madison is perfectly competitive. The market demand for plastic chairs is given as $P = 130 - Q$. The market supply for plastic chairs is given as $P = 2 + Q$. Each firm faces the cost functions $TC = 4q^2 + 2q + 64$ and $MC = 8q + 2$.

a.) Determine the equilibrium quantity and price for this plastic chair market.

Solution: Equating market demand and market supply,

$$\begin{aligned} 130 - Q &= 2 + Q \\ \implies Q &= 64, P = 66 \end{aligned}$$

b.) What are the break-even price and the shut-down prices for a representative firm in the short run?

Solution:

First we find the break-even price.

$$\begin{aligned} ATC = MC &\iff 4q + 2 + 64/q = 8q + 2 \\ q &= 4, P = 34 \end{aligned}$$

Now, we find the shut-down price.

$$\begin{aligned} AVC = MC &\iff 4q + 2 = 8q + 2 \\ q &= 0, P = 2 \end{aligned}$$

c.) At the current equilibrium price, what is the quantity of chairs provided by a representative firm? Calculate a representative firm's profit.

Solution: We solved that $P = 66$. Using the firm supply curve, $MC = P = 8q + 2$, we have

$$\begin{aligned} 66 &= 8q + 2. \\ \implies q &= 8. \end{aligned}$$

Profit is found by calculating $TR - TC$. For a firm, $TR = 8 \times 66 = 528$. We use the total cost function, $TC = 4(8)^2 + 2(8) + 64 = 336$. Thus, profit = $\pi = 192$.

d.) How many firms are in the market in the short run?

Solution: With market quantity $Q = 64$ and firm quantity $q = 8$, there must be $Q/q = 8$ firms.

e.) What is the long-run profit maximizing level of output for a representative firm? What is the long-run profit?

Solution: In the long-run, the equilibrium price must be the break-even price, $P = 34$. Every firm will produce $q = 4$ (from part b). All firms earn zero profit.

f.) What is the long-run equilibrium price in the market?

Solution: $P = 34$.

g.) How many firms are in the market in the long run?

Solution: Each firm must produce $q = 4$. With a market price of $P = 34$, we can find the market quantity by plugging this break-even (and therefore long run equilibrium price) into the market demand. Therefore, $Q = 96$. Now $\#firms = 96/4 = 24$.

Exercise 4 If profits are negative in the short run in a perfectly competitive industry, which of the following would you not expect to happen as the market moves to the long run (assuming no external economies or external diseconomies of scale)?

- The market price will increase.
- Firms will exit the market.
- Total market output will fall.
- Each firm's individual demand curve will shift down.

Solution: d.) In the long run, the price must rise. Each firm's demand curve is perfectly elastic at the market price. Therefore, the firm's demand curve will shift up.

Exercise 5 Consider the following information for a T-shirt manufacturing firm that can sell as many T-shirts as it wants for \$3 per shirt.

| Number of Workers | Quantity of Shirts | MPL | TR | MRPL |
|-------------------|--------------------|-----|----|------|
| 0 | 0 | | | |
| 1 | 30 | | | |
| 2 | 80 | | | |
| 3 | 110 | | | |
| 4 | 135 | | | |
| 5 | | 20 | | |
| 6 | 170 | | | |
| 7 | | | | 30 |
| 8 | | | | 15 |

- a.) Fill in all the blanks in the table.
- b.) Verify that MRPL for this firm can be calculated in two ways: (1) change in the TR from adding another worker and (2) MPL times the price of the output.
- c.) If this firm must pay a wage rate of \$40 per worker per day, how many workers should be hired now? Why?
- d.) Suppose the wage rate rises to \$50 per worker. How many workers should be hired now?
- e.) Suppose the firm adopts a new technology that doubles output at each level of employment and the price of shirts remains at \$3. What is the effect of this new technology on MPL and MPRL? At a wage of \$50, how many workers should the firm hire now?

Solution:

| <i>Number of Workers</i> | <i>Quantity of Shirts</i> | <i>MPL</i> | <i>TR</i> | <i>MRPL</i> |
|--------------------------|---------------------------|------------|-----------|-------------|
| 0 | 0 | - | - | - |
| 1 | 30 | 30 | 90 | 90 |
| 2 | 80 | 50 | 240 | 150 |
| 3 | 110 | 30 | 330 | 90 |
| 4 | 135 | 25 | 405 | 75 |
| 5 | 155 | 20 | 465 | 60 |
| 6 | 170 | 15 | 510 | 45 |
| 7 | 180 | 10 | 540 | 30 |
| 8 | 185 | 5 | 555 | 15 |

When the wage is \$40, the firm should hire 6 workers. When the wage is \$50, the firm should hire 5 workers.

With the new technology, the MPL and therefore MRPL double. Now, the firm should hire 7 workers.